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On the spectra of scalar mesons from HQCD models

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Abstract: We determine the holographic spectra of scalar mesons from the fluctuations of the embedding of flavor D-brane probes in HQCD models. The models we consider include a generalization of the Sakai Sugimoto model at zero temperature and at the "high-temperature intermediate phase", where the system is in a deconfining phase while admitting chiral symmetry breaking and a non-critical 6d model at zero temperature. All these models are based on backgrounds associated with near extremal N_c D4 branes and a set of $N_f \ll N_c$ flavor probe branes that admit geometrical chiral symmetry breaking. We point out that the spectra of these models include a 0^{--} branch which does not show up in nature. At zero temperature we found that the masses of the mesons M_n depend on the "constituent quark mass" parameter m_q^c and on the excitation number n as $M_n^2 \sim m_q^c$ and $M_n^2 \sim n^{1.7}$ for the ten dimensional case and as $M_n \sim m_q^c$ and $M_n \sim n^{0.75}$ in the non-critical case. At the high temperature intermediate phase we detect a decrease of the masses of low spin mesons as a function of the temperature similar to holographic vector mesons and to lattice calculations.

Keywords: AdS-CFT Correspondence, Brane Dynamics in Gauge Theories, Gauge-gravity correspondence.

Contents

Whereas realizing confinement in dual holographic models of QCD (HQCD) is easy, the incorporation of flavored chiral quarks and in particular chiral symmetry breaking is more difficult. Sakai and Sugimoto [1] proposed a model that admits the two phenomena. It is based on placing a set of N_f D8 and anti D8 probe flavor branes into the gravity model of near extremal D4 branes [2, 3].

The mesonic spectra is one of the most important properties of hadron dynamics that can be "measured" in the HQCD laboratory. The low spin mesons are associated with the fluctuations of the fields that reside on the probe flavor branes, the vector mesons with the $\mathrm{U}(N_f)$ flavor gauge fields and the scalar mesons with the embedding of the probe branes.[1] Here in this paper we focus only on scalar mesons. The motivation behind addressing this problem are the following:

(i) To verify that the meson spectrum at zero temperature does not include tachyonic modes. Had there been such modes it would have indicated that the system is unstable. Since the model of [1] is based on placing branes and anti-branes one may be worried that the system is unstable and hence the importance of this verification.

(ii) The spectrum of the scalar mesons has been determined already in [1]. However the attempts to derive it in generalizations of the model where the asymptotic separation of the brane anti-brane L is smaller than half of the circumference of the compactified direction x_4, namely for $L \leq \pi R$ failed for the symmetric modes [5, 6]

[1] High spin mesons are naturally described by semi-classical spinning string configurations [4]

(iii) To determine the dependence of the spectrum on the excitation number n and the parameter m_q^c defined in (3.6) that is related to the constituent quark mass. In addition one naturally would like to compare the explicit ratios of meson masses that one deduces from any given HQCD model and the experimental data to get an indication of how well the model describes real hadron physics.

(iv) To further examine the differences of physical properties extracted from critical models to non-critical models which were previously discussed in [5, 7, 8]. The spectrum of scalar mesons was extracted also in other HQCD models [9–11]. For further reading see [12] and references therein.

We can summarize the outcome of the paper as follows

- We were able to choose coordinates that avoid the singularities that were encountered in previous works [5, 6] and determine the spectrum of both the anti-symmetric as well as symmetric branches.
- We found that in the models examined and in particular the original model of [1] the symmetric solutions correspond to scalar mesons of the form 0^{++} whereas the anti-symmetric solutions correspond to 0^{--} mesons. This property which seems to be in common to a HQCD models based on probe branes and anti-branes, contradict the low lying spectrum in nature, there are no low lying 0^{--} mesons.
- At zero temperature we found that the masses of the mesons M_m depend on the "constituent quark mass" m_q^c and on the excitation number n as $M_m^2 \sim m_q^c$ and $M_m^2 \sim n^\alpha$ with $\alpha \sim 1.7$ for the ten dimensional case and as $M_m \sim m_q^c$ and $M_m \sim n^\beta$ with $\beta \sim 0.75$ when a CS term is incorporated and $\beta \sim 1$ without such a term in the non-critical case. At the high temperature intermediate phase we detect a decrease of the masses of low spin mesons as a function of the temperature similar to holographic vector mesons and to lattice calculations.

The paper is organized as follows. We begin in section 1 with a brief review of the holographic models we investigate. We summarize the main features of the model of Sakai and Sugimoto at zero and finite temperature and an analogous six dimensional non-critical model. In section 2 we describe the extraction of scalar mesons from the fluctuations of the embedding. In particular we point out that in the coordinates introduced in [1] the eigenvalue problem admits a singularity that prevents the numerical determination of the eigenvalues. A different coordinate system is presented in section 3 which evades the problem of the singularity. Using these coordinates, the spectrum of masses as a function of the constituent mass and excitation number is derived. The spectrum of scalar mesons that follows from a non critical model of near extremal D4 branes is analyzed in section 4. Section 5 addresses the issue of parity and charge conjugation of the scalar mesons. It is pointed out that the spectrum includes 0^{--} mesons which do not show up in nature. Section 6 is devoted to the spectrum of mesons above the deconfining phase transition in the "intermediate phase". We then summarize the results and raise certain open questions.

1. Review of the holographic models

1.1 The Sakai Sugimoto model

The model of [2], describes the near horizon limit of N_c $D4$-branes wrapping a circle in the x_4 direction with anti periodic boundary condition for the fermions. Into this background a stack of N_f $D8$ is placed at $x_4 = 0$ and a stack of N_f $\bar{D}8$ is at the anti-podal point of the x_4 circle [1]. Assuming $N_f \ll N_c$ one can overlook the modification of the metric and dilaton due to the backreaction of the background by the N_f $D8$-$\bar{D}8$ systems and continue to use the metric and dilaton associated with the N_c $D4$ alone. Therefore the metric, dilaton and the RR four form are given by

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{3/2}\left[-dt^2+\delta_{ij}dx^i dx^j + f(u)dx_4^2\right]+\left(\frac{R_{D4}}{u}\right)^{3/2}\left[\frac{du^2}{f(u)}+u^2 d\Omega_4^2\right] \tag{1.1}$$

$$F_4 = \frac{2\pi N_c}{V_4}\epsilon_4\,, \quad e^{\phi} = g_s\left(\frac{u}{R_{D4}}\right)^{3/4}, \; R_{D4}^3 = \pi g_s N_c l_s^3\,, \quad f(u) = 1-\left(\frac{u_\Lambda}{u}\right)^3$$

Where V_4 denotes the volume of the unit sphere Ω_4 and ϵ_4 its corresponding volume form. l_s is the string length and g_s a parameter related to the string coupling. The x_4 is the compactified direction that is asymptotically transverse to the $D8$. The manifold spanned by the coordinate u, x_4 has the topology of a cigar where its tip is at the minimum value of u which is $u = u_\Lambda$. The periodicity of this cycle is uniquely determined to be

$$\delta x_4 = \frac{4\pi}{3}\left(\frac{R_{D4}^3}{u_\Lambda}\right)^{1/2} = 2\pi R \tag{1.2}$$

in order to avoid a conical singularity at the tip of the cigar. The classical profile of the $D8$ probe brane in this background is given by the classical solution to the e.o.m of the DBI action of that probe brane. The $D8$ DBI action is

$$S_{D8} = T_8\int dt d^3x du d\Omega_4 e^{-\phi}\sqrt{-det\hat{g}} = \tilde{T}_8\int dt d^3x du u^4\sqrt{f(u)(\partial_u x_4)^2+\frac{R_{D4}^3}{u^3 f(u)}} \tag{1.3}$$

$$= \tilde{T}_8\int dt d^3x dx_4 u^4\sqrt{f(u)+\left(\frac{R_{D4}}{u}\right)^3\frac{u'^2}{f(u)}} \tag{1.4}$$

where $\hat{g}$ stands for the pullback metric on the $D8$ brane. The simplest way of solving this e.o.m is by noting that the action is independent of x_4 and so its Hamiltonian is conserved,

$$\frac{u^4 f(u)}{\sqrt{f(u)+\left(\frac{R_{D4}}{u}\right)^3\frac{u'^2}{f(u)}}} = u_0^4\sqrt{f(u_0)} = const \tag{1.5}$$

where we assumed that there is a point u_0 where the curve $u(x_4)$, which describes the profile of the $D8$ brane in the (u, x_4) plane, has a minimum. At that point the $D8$ brane bends, namely the $D8$-$\bar{D}8$ join together. After some algebra one finds

$$\left(\frac{\partial x_4}{\partial u}\right)_{\rm cl} = \frac{1}{f(u)(\frac{u}{R_{D4}})^{3/2}\sqrt{\frac{f(u)u^8}{f(u_0)u_0^8}-1}} \tag{1.6}$$

Hence we find that the profile of the $D8$ brane probe is

$$x_4(u) = \int_{u_0}^{u} \frac{du}{f(u)(\frac{u}{R_{D4}})^{3/2}\sqrt{\frac{f(u)u^8}{f(u_0)u_0^8} - 1}} \tag{1.7}$$

where u_0 is a constant of integration setting the lowest value of u to which the $D8$ brane is extending. At that point the $D8$ brane joins the $\bar{D}8$ brane and the brane is extended back into the UV. The value of u_0 also sets the asymptotic distance L between the position of the $D8$ and $\bar{D}8$ brane

$$L = \int dx_4 = 2\int_{u_0}^{\infty} \frac{du}{u'} = 2(\frac{R_{D4}^3}{u_0}\Big)^{1/2} \int_1^{\infty} dy \frac{y^{-3/2}}{\sqrt{f(y)}\sqrt{\frac{f(y)}{f(1)}y^8 - 1}} \tag{1.8}$$

Hence we see

$$L \propto \left(\frac{R_{D4}^3}{u_0}\right)^{1/2} \tag{1.9}$$

For later use we define

$$\gamma = \frac{u^8}{f(u)u^8 - f(u_0)u_0^8} \tag{1.10}$$

The DBI action then becomes

$$S = T_8 \int e^{-\phi}\sqrt{|det\hat{g}_0|} \sim \int d^4x du \gamma^{1/2} u^{5/2}$$

1.2 Thermodynamics of the Sakai Sugimito model

In [13][2] a study of the thermodynamics of the Sakai Sugimito model was carried out using the conjecture presented in [2].

The conjecture states that the thermodynamics of a field theory with a gravitational dual is determined by taking into account the contribution to the saddle point approximation from all the gravitational backgrounds with the correct 'UV' asymptotic, with compactified Euclidean time direction of period $\beta = \frac{1}{T}$ and with anti-periodic boundary conditions for the fermions along this direction. The temperature of the field theory is $T = 1/\beta$ and its properties are read from the manifold responsible for the most dominant contribution to the saddle point approximation, namely the one that has the lowest free energy.

Whenever one background looses its domination to another background as we vary the temperature, a phase transition occurs in the dual field theory.

In [13] two manifolds were found to have the same 'UV' asymptotic as the one of Sakai and Sugimoto model, the background (1.1), and the same configuration only with the time and x_4 directions interchange.

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{3/2}[-f(u)dt^2 + \delta_{ij}dx^i dx^j + dx_4^2] + \left(\frac{R_{D4}}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right] \tag{1.11}$$

[2]see also [14].

with

$$f(u) = 1 - \left(\frac{u_T}{u}\right)^3 \tag{1.12}$$

and the temperature is given by

$$\delta t = \frac{4\pi}{3}\left(\frac{R_{D4}^3}{u_T}\right)^{1/2} = \beta \tag{1.13}$$

The difference between the free energy densities of the two backgrounds is proportional to $N_c^2[(2\pi T)^6 - 1/R^6]$.[3] This means that when the circumference of the x_4 cycle is smaller than that of the time direction, namely when $T < 1/2\pi R$ the background (1.1) is the dominant one, while when the opposite occurs and $T > 2\pi R$ the action of (1.11) will dominate. At the temperature $T = 1/2\pi R$ the two actions are the same since the two backgrounds are different by the labeling of the coordinates, so at $T = T_c = 1/2\pi R$ the system has a first order phase transition. In [13] it was argued that in the dual field theory, the physical interpretation of this phase transition is a transition from a confined phase at $T < 1/2\pi R$ to a deconfined one at $T > 1/2\pi R$. This can be seen via a computation of the quark anti-quark potential [15] in the two backgrounds. Another indication to this interpretation is that the renormalized free energy of the low temperature phase shows a N_c^0 behavior while that of the high temperature phase shows a N_c^2 one. Hence from now on we will denote $T_c = T_d$.

At the high temperature phase there is another possible classical solution to the profile of the $D8$ brane which is a configuration with constant x_4, namely $x_4(u) = 0, L$.[4]

Now since the bulk free energy is the same for the two configurations of the $D8$ branes, the difference between the free energy of the $D8$ probes determines which of the two configurations is the preferable one for a given temperature. It turns out that the transition between the two configuration depends on the parameter $y_T = \frac{u_0}{u_T}$, and its value at the phase transition turns out to be $y_T^c \sim 0.73572$.

Using eq. (1.8) we find $L_c = 0.751\left(\frac{R_{D4}^3}{u_0}\right)^{1/2}$, hence at the critical point $y_T = y_T^c$ the critical temperature is set by the asymptotic distance between the branes (setting $R_{D4} = 1$)

$$T_c = \frac{3}{4\pi}u_T^{1/2} = \frac{3}{4\pi}(y_T^c u_0)^{1/2} = 0.154/L \tag{1.14}$$

The field theory sees this transition as chiral symmetry restoration at high temperature. This interpretation is natural since the $D8$ branes are now disconnected and there is an $\mathrm{U}(N_f) \times \mathrm{U}(N_f)$ global symmetry.

Hence we will denote this critical temperature as $T_{\chi SB}$. Note that this only happens at the high temperature phase so there is still the condition $T_{\chi SB} = 0.154/L > 1/2\pi R$.

[3] Of course in our model there are also $D8$ brane which their DBI action will contribute to the total free energy of the configuration as well, but this is sub-leading to the bulk action since the bulk action is of order N_c^2 and the contribution of the $D8$ is of order $N_c \cdot N_f$ which is negligible in the probe approximation

[4] This configuration was not possible in the low temperature, but in the high temperature phase the time circle shrink to zero at $u = u_\Lambda$ and so the $D8$ brane can just smoothly end there.

So if $L > 0.97R$, we find that T_d is always higher than $T_{\chi SB}$, and so deconfinement and chiral symmetry restoration phase transition happen together. We see that in this model χSB and confinement appear independently of one another as a result of the existence of the free parameter L coming from the $5d$ nature of the field theory.

1.3 Non critical holographic model

A non critical model with very similar properties to the Sakai-Sugimoto model was presented in [5, 7],[5] this model consists of a non-extremal configuration of N_c $D4$ branes placed in a six dimension space-time with one of the $D4$ coordinates taken to be periodic with anti periodic boundary conditions for the fermions.

The metric, dilaton and RR six-form field take the form [7]

$$\begin{aligned} ds^2 &= \left(\frac{u}{R_{\text{AdS}}}\right)^2 dx_{1,3}^2 + \left(\frac{R_{\text{AdS}}}{u}\right)^2 \frac{du^2}{f(u)} + \left(\frac{u}{R_{\text{AdS}}}\right)^2 f(u) dx_4^2 \\ F_{(6)} &= Q_c \left(\frac{u}{R_{\text{AdS}}}\right)^4 dx_0 \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge du \wedge dx_4 \\ e^{\phi} &= \frac{2\sqrt{2}}{\sqrt{3}Q_c}; \qquad R_{\text{AdS}}^2 = \frac{15}{2} \end{aligned} \tag{1.15}$$

with

$$f(u) = 1 - \left(\frac{u_\Lambda}{u}\right)^5 \tag{1.16}$$

where Q_c is proportional to N_c, the number of color $D4$ branes. In order to avoid conical singularity the periodicity of the cycle of x_4 is set to be:

$$x_4 \sim x_4 + \delta x_4 \,; \qquad \delta x_4 = \frac{4\pi R_{\text{AdS}}^2}{5u_\Lambda} \tag{1.17}$$

Of course the curvature of order one of this background makes the leading order supergravity an unjustified approximation to string theory on this background. Nevertheless its believed that at least the extremal model due to its symmetries, is indeed a good background for the study of non-critical string theory [16]. Now we place N_f $D4$ branes which are transverse to the S^1 cycle and extend up to infinity in the u direction. The properties of the four dimensional low energy effective field theory living on the intersection of these color and flavor $D4$ then seems to be very similar to those found at the Sakai Sugimoto model. Thus we would like to study its spectrum of scalar excitations and check if there is no tachyon in the model.

Just like in the critical model the $D4$ brane may bend on the (u, x_4) cigar and in order to find its profile one must solve the e.o.m of the x_4 coordinate. This e.o.m is derived from the action of the flavor $D4$ branes, namely the DBI action plus the CS term which are given by

$$S_{D4} = -T_4 \int d^5x e^{-\phi} \sqrt{-\det(\hat{g})} + T_4 \tilde{a} \int P(C_{(5)}) \tag{1.18}$$

[5] For other non-critical SUGRA models with flavor see [16 – 20].

Following similar steps to those taken in the previous section we find

$$x_{4,cl}(u) = \int_{u_0}^{u} \frac{(u_0^5 f^{1/2}(u_0) - a u_0^5 + a u'^5) du'}{(\frac{u'}{R_{\rm AdS}})^2 f(u') \sqrt{u'^{10} f(u') - (u_0^5 f^{1/2}(u_0) - a u_0^5 + a u'^5)}} \tag{1.19}$$

where $a = \frac{2}{\sqrt{5}}$.

2. Fluctuation of the embedding and scalar mesons

We now turn our attention to the study of the fluctuation of the $D8$ brane around its classical profile. As was mentioned in the introduction, one has a twofold interest in these fluctuations:

(i) They correspond to scalar mesons in the dual gauge theory.

(ii) Tachyonic modes of the fluctuation signals an instability of the system.

We start by expanding the x_4 coordinate around its classical value and define the fluctuation $\xi(u, x^\mu)$ as follows:

$$x_4(u, x^\mu) = x_4(u)_{\rm cl} + \xi(u, x^\mu) \tag{2.1}$$

Substituting this into the action (1.3) and expanding to quadratic order in ξ we find the following action for the fluctuations

$$S \propto \frac{1}{2} \int d^4x du \Big\{ u^{5/2} R_{D4}^3 \gamma^{-1/2} \eta^{\mu\nu} \partial_\mu \xi \partial_\nu \xi + u^{11/2} \gamma^{-3/2} (\partial_u \xi)^2 \Big\} \tag{2.2}$$

where γ is defined in (1.10). We now introduce the following mode expansion

$$\xi(u, x^\mu) = \sum_{n=0}^{\infty} f_n(x^\mu) \xi_n(u) \tag{2.3}$$

Using the symmetries along the x^μ directions we have

$$\eta^{\mu\nu} \partial_\mu \partial_\nu f_n = -m_n^2 f_n \tag{2.4}$$

The e.o.m for the ξ_n modes reads

$$\partial_u [(u^{11/2} \gamma^{-3/2}) \partial_u \xi_n] = -m_n^2 R_{D4}^3 u^{5/2} \gamma^{-1/2} \xi_n \tag{2.5}$$

or in its canonical form

$$\left\{ \partial_u^2 + \left[\left(\frac{12}{u} - \frac{15}{2u^4} \right) \gamma - \frac{13}{2u} \right] \partial_u \right\} \xi_n = -\frac{m_n^2 R_{D4}^3 \gamma}{u^3} \xi_n \tag{2.6}$$

For $u_0 \gg u_\Lambda$, $f(u) \to 1$, the e.o.m simplifies and the qualitative behavior of m_n can be determined by using dimensional arguments [13]. Define the dimensionless parameter $v = \frac{u}{u_0}$ then for the limit $u_0 \gg u_\Lambda$ where $f \to 1$

$$\gamma \to \frac{1}{1 - \frac{1}{v^8}} \tag{2.7}$$

The e.o.m in terms of v reads

$$\partial_v(v^{11/2}\gamma^{-3/2})\partial_v\xi_n = -m_n^2\frac{R_{D4}^3}{u_0}v^{5/2}\gamma^{-1/2}\xi_n \tag{2.8}$$

Since the L.H.S is dimensionless so must be the R.H.S and hence

$$m_n^2 \propto \frac{u_0}{R_{D4}^3} \tag{2.9}$$

Using the relation (1.9) between u_0 and L we find

$$m_n \propto \frac{1}{L} \tag{2.10}$$

while the mass of the glueball is related to $m_{\rm gb} \sim \frac{1}{R}$. For the case $u_\Lambda = u_0, \quad L = \pi R$ so the glueball and mesons masses have the same scale. However in the general case where $u_0 > u_\Lambda$ there are two different scales $m_n \sim \frac{1}{L} > \frac{1}{R} \sim m_{\rm gb}$.

In order to find the exact spectrum of the eigenvalues of (2.6) one can use the 'shooting' technique which is implemented by solving the eigenvalue problem as a second order o.d.e with boundary conditions given in two different points (a two point boundary problem). This is possible if there are two different points on the grid in which we know ξ and ξ'. Then ξ and ξ' at one of these two points may serve as boundary conditions from which we can integrate from and try to match it to some single boundary condition combined from ξ and ξ' on the other point,[6] of course second order eq. can only satisfy two boundary condition so the matching is only possible for the correct eigenvalue! Since the correct eigenvalue is not previously known one shoots with different eigenvalues until the matching is obtained.

In our case we can find the boundary value at the asymptotic $z \to \infty$ using the asymptotic expansion of the normalizable solution of eq. (2.6) at $z \to \infty$.[7]

The second point would be $z = 0$ $(u = u_0)$, since (2.6) is symmetric under $z \to -z$ we can split the spectrum for symmetric and anti-symmetric modes under this reflection. This splitting of the spectrum will be of great phenomenological importance when we shall discuss the parity and charge conjugation of these modes as scalar mesons. In practice the boundary condition that preserve only the symmetric mode is $\partial_z\xi_n(z=0) = 0$, and for anti-symmetric ξ we shall demand the boundary condition $\xi_n(z=0) = 0$.

However there is a problem with these coordinates at $u = u_0$ since, $\frac{dx_{4,cl}}{du}|_{u=u_0} \to \infty$ (see eq. (1.6)). An odd perturbation to the classical configuration will cause no change in the shape of this singularity but an even one will, and so will also have a singular derivative.

This problem is reflected in the singularity of the e.o.m (2.6) at $u \to u_0$. To see this behavior explicitly we change coordinates to a dimensionless parameter z as follows

$$u^3 = u_0^3 + u_\Lambda^3 z^2 \tag{2.11}$$

[6]Since the o.d.e is linear the two boundary values can differ by a different normalization of ξ so in practice one should use a clever boundary condition at the second point such that it is invariant to the normalization, for instance one can use ξ'/ξ.

[7]The asymptotic solutions for the e.o.m are obtained by series expansion at $z \gg 1$, for $u_0 = u_\Lambda$ the first order solutions are $\xi_n \sim O(1)$ and $\xi_n \sim O(z^{-1})$ [1], the first in a non-normalizable one while the other is. The same is true in our case where $u_0 > u_\Lambda$.

the eigenvalue problem (2.6) then becomes

$$\left\{\partial_z^2+\left[\frac{5u_\Lambda^3 z}{u_0^3+u_\Lambda^3 z^2}-\frac{1}{z}-\frac{\gamma' u_\Lambda^3 z}{(u_0^3+u_\Lambda^3 z^2)^{2/3}\gamma}\right]\partial_z\right\}\xi_n=-\frac{m_n^2 R_{D4}^3 u_\Lambda^6\gamma z^2}{(u_0^3+u_\Lambda^3 z^2)^{7/3}}\xi_n \tag{2.12}$$

where γ' stands for the derivative of γ with respect to u. Since

$$\gamma_{z\to 0}=\frac{\frac{3u_0^6}{u_\Lambda^3(8u_0^3-5u_\Lambda^3)}}{z^2}\,;\quad \gamma'_{z\to 0}=-\frac{\frac{9u_0^8}{u_\Lambda^6(8u_0^3-5u_\Lambda^3)}}{z^4} \tag{2.13}$$

we find that this equation has a regular singularity at $z=0$!

Indeed it was already noticed in [6] that by employing the 'shooting' technique only half of the spectrum could be found, namely only the odd modes were seen while the even ones could not be obtained, these modes that should have been obtained by integrating the normalizable solution ($\xi_n\sim\frac{1}{z}$) from the asymptotic the symmetric boundary conditions to

$$\partial_z\xi_n(z=0)=0. \tag{2.14}$$

turned out to be singular and could not be integrated. In [1] only the special case of $u_0=u_\Lambda$ was analyzed, in this case since $\lim_{u_0\to u_\Lambda}\partial_u x_{\rm cl}=0$, a smooth and nonsingular transformation into cartesian coordinates is allowed via

$$u^3=u_\Lambda^3+u_\Lambda^3(z^2+y^2)\,;\quad x_4=R\arctan\left(\frac{y}{z}\right) \tag{2.15}$$

The corresponding action for y is (after setting $u_\Lambda=1$)

$$S\sim\int d^4x dz\left[\frac{(\partial_\mu y)^2}{u(z)}+u(z)^3(\partial_z y)^2+2y^2\right] \tag{2.16}$$

inserting the expansion $y=\sum_{n=1}\varphi_n(x^\mu)y_n(z)$ the e.o.m for y_n is

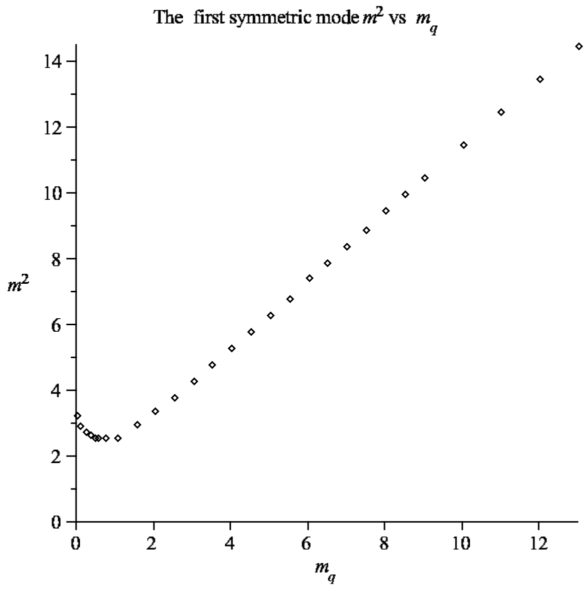
$$\partial_z^2 y_n+\frac{2z}{1+z^2}\partial_z y_n-\frac{2y_n}{1+z^2}=\frac{m_n^2}{(1+z^2)^{4/3}}y_n \tag{2.17}$$

which is non-singular. For the more general case of $u_0>u_\Lambda$ we were unable to find a similar coordinate transformation and hence we follow a different approach described in the next section.

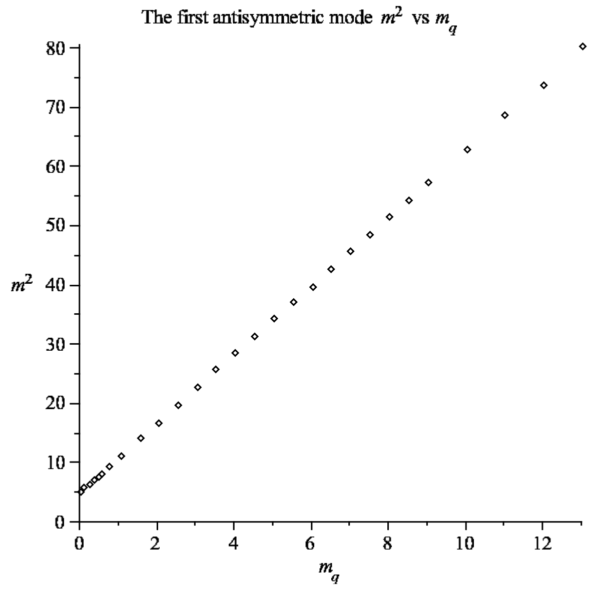
3. A regular e.o.m for the scalar fluctuation at the low temperature phase

As we have seen above, we could not obtain the even modes of the fluctuation[8] around the classical curve because $\frac{dx_{4,cl}}{du}$ diverges at $u=u_0$. The issue of choosing a direction along which one should analyze the fluctuations, has been discussed in the context of the stringy description of the Wilson line [21]. It was found that the safest approach is to use the fluctuation in the direction which is normal to the classical configuration. For our case the

[8]If the classical curve $x_{4,cl}$ was odd, then the odd mode would become singular.



(A)



(B)

Figure 1: (A) The mass squared m_1^2 of the lowest excited symmetric mode as a function of m_q^c $(R_{D4} = u_\Lambda = 1)$

Figure 2: (B) The mass squared m_2^2 of the lowest excited antisymmetric mode as a function of m_q^c $(R_{D4} = u_\Lambda = 1)$

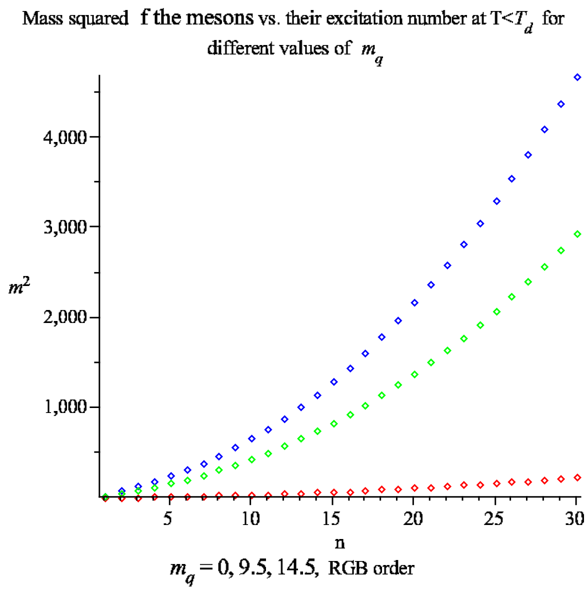


Figure 3: (A) The tower of the mesons squared mass m_n^2 in the low temperature phase $(R_{D4} = u_T = 1)$

normal to the classical configuration at the tip $u = u_0$ is along the u direction. Thus from here on we study the fluctuation in the u direction, that is

$$u(x_4, x^\mu) = u_{\rm cl}(x_4) + \xi(x_4, x^\mu) \tag{3.1}$$

our classical curve would be $u_{\rm cl}(x_4)$ and as can be seen from (1.6) we have $\frac{du_{\rm cl}}{dx_4}|_{x=0} = 0$ so the point $u(x_4 = 0) = u_0$ poses no problem now! The quadratic action for these fluctuations is (after setting $u_\Lambda = 1$)

$$S = \frac{1}{2}\int dx_4 \Big\{ \frac{a_0}{u^{11}f^3}(\partial_{x_4}\xi)^2 + \frac{1}{u^3 f}(\partial_\mu \xi)^2 - \frac{(11u^{14} + 18a_0 + 3u^{11} - 12u^8 - 27a_0(u^3 + u^6) - 2u^5)}{2u^{16}f^3}\xi^2 \Big\} \tag{3.2}$$

where $a_0 = u_0^8 f(u_0)$ and it should be understood that $u = u_{\rm cl}(x_4)$ and its formal expression is

$$u(x_4) = \int_0^{x_4} dx_4 f(u)(\frac{u}{R_{D4}})^{3/2}\sqrt{\frac{f(u)u^8}{f(u_0)u_0^8} - 1} \tag{3.3}$$

after plugging a mode expansion the e.o.m in its canonical form is

$$\partial_x^2 \xi_n - \left(\frac{11}{u} + \frac{9}{uf}\right)u_x \partial_x \xi_n - \frac{f^2 u^8 m_n^2}{a_0}\xi_n + \frac{(11u^{14} + 18a_0 + 6u^{11} - 12u^8 - 27a_0(u^3 + u^6) - 2u^5)}{2a_0 u^5}\xi_n = 0 \tag{3.4}$$

where $u_x = \partial_{x_4} u_{\rm cl}$. Since there is no analytic expression for the integral in (3.3), we obtained $u(x_4)$ numerically during the integration of eq. (3.4) when 'shooting' to find the eigenvalues of (3.4).

The resulted spectra are summarized in figures 1, 2 and 3. The following properties characterize these spectra

- The first observation one can make is that for $u_0 = u_\Lambda$ our results for the symmetric and anti-symmetric lowest lying states match those of [1].

$$m_s^2 = 3.3\,; \qquad m_{\rm as}^2 = 5.3 \tag{3.5}$$

- The figures 1 and 2 describe the dependence of the squared mass of the first excited symmetric and anti-symmetric states as a function of the "constituent quark mass" defined in [5] and [4], as follows

$$m_q^c = \frac{1}{2\pi\alpha'}\int_{u_\Lambda}^{u_0}\sqrt{-g_{tt}g_{uu}}du = \frac{1}{2\pi\alpha'}\int_{u_\Lambda}^{u_0} f^{-1/2}(u)du \tag{3.6}$$

  This parameter relates to the constituent quark mass and not to the current algebra (QCD) mass, since even when it is turned on the fluctuations that correspond to the pions are massless. In fact the quantity dual to the constituent quark mass should associate with m_q^c plus a constant term which is independent of u_0 since already for $u_0 = u_\Lambda$ the mesons are massive and hence there is a non-trivial constituent mass. This assignment is also in agreement with the semi-classical description of high spin mesons [4] and their stringy split into two lower mass mesons [22]. ¿From these

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figures we see that indeed for $u_0 > u_\Lambda$ the square of the mass of the scalars grows linearly with m_q^c. This is to be contrasted with the results found in [5] for vector mesons of non-critical models where the mass itself is found to be linear with the m_q^c (see also down in section 5.)

- We have also determined the spectrum of the higher excited mesons, both the symmetric as well as the anti-symmetric ones. The dependence of the squared masses on the excitation number for various values of m_q^c is drawn in figure 3. The linear fits to these curves are given by

$$\begin{aligned} m_n^2 &= 3.3 + 1.6 n^{1.7} & m_q^c &= 0 \\ m_n^2 &= 10.5 + 6.5 n^{1.789} & m_q^c &= 9.3 \\ m_n^2 &= 15.8 + 9.5 n^{1.818} & m_q^c &= 14.3 \end{aligned} \tag{3.7}$$

  Stringy modes are characterized by the well known $m^2 \sim n$ behavior. We thus see that the scalar meson spectra that follows from the model of [1] do not correspond to stringy modes. This is of course of no surprise since it follows from a low energy effective field theory and not from a semi-classical treatment.

- Last by not least we see from figure 1 that the lowest scalar excitation remain non-tachyonic for all values of u_0 which serves as partial evidence for the stability of the Sakai Sugimoto model.

4. Scalar mesons in a non critical holographic model

We would like now to find the masses of the scalar modes associated with the fluctuations of the probe brane around the classical profile in the non-critical gravity background of [7]. Using the background (1.15) in an effective action that includes the DBI plus the CS term

$$S_{\rm CS} \sim \int_{D4} C_5 = \int_{D4} \frac{u^5}{R_{\rm AdS}^4} \tag{4.1}$$

we can easily find that the profile of the D4 prob branes is given by

$$x_{4,cl}(u) = \int_{u_0}^{u} du' \frac{u_0^5 f^{1/2}(u_0) + \frac{2}{\sqrt{5}}(u'^5 - u_0^5)}{\left(\frac{u'}{R_{\rm AdS}}\right)^2 f(u') \sqrt{u'^{10} f(u') - (u_0^5 f^{1/2}(u_0) + \frac{2}{\sqrt{5}}(u'^5 - u_0^5))^2}} \tag{4.2}$$

substitution a fluctuation of the form (3.1) for x_4 and expanding the action up to linear and quadratic term, we find the e.o.m for the fluctuation modes. However, it was found in [8] that including the CS does not yield a sensible thermal phase diagram and hence we discuss separately an effective action that includes only a DBI action and one with both the DBI and CS terms. We start first with the former case: Analyzing the spectrum in a similar manner to the analysis of section 3 we find that the fluctuations are subjected to the following eigenvalue equation

$$\partial_u (u^4 \gamma^{-3/2}) \partial_u \xi_n = -\frac{R_{\rm AdS}^4 m_n'^2 u^2}{\gamma^{1/2}} \xi_n \tag{4.3}$$

Like in the critical case, for $u_0 \gg u_\Lambda$ the qualitative behavior of m'_n can be seen by changing the variable u into the dimensionless parameter $y = \frac{u}{u_0}$. At the limit $u_0 \gg u_\Lambda$ we find that $f(u) \to 1$ and so

$$\gamma \to \frac{1}{u_0^2(y^2 - \frac{1}{y^8})} \tag{4.4}$$

and find that in terms of the dimensionless parameter y the e.o.m is now

$$\partial_y(y^4\gamma^{-3/2})\partial_y\xi_n = -\frac{R_{\text{AdS}}^4 m_n'^2 y^2}{u_0^2\gamma^{1/2}}\xi_n \tag{4.5}$$

Since the L.H.S is dimensionless so is the R.H.S and we find

$$m_n'^2 \propto \frac{u_0^2}{R_{\text{AdS}}^4} \tag{4.6}$$

Note that due to the different background now $L \sim \frac{R_{\text{AdS}}^2}{u_0}$ and hence again we get that $m'_n \sim \frac{1}{L}$. However in terms of m_q^c the asymptotic behavior is $m'_n \sim m_q^c$ and not ${m'_n}^2 \sim m_q^c$ as was the case for the mesons of the critical model.

Repeating the exact same steps as for the critical case we find that the quadratic action for fluctuation in the x_4 direction around the classical curve leads to an e.o.m which is singular at $u = u_0$ and as a consequence the attempt carried in [5] to obtain the spectrum of the even modes had indeed failed. And so like in the critical case we turn to study the fluctuation in the u direction instead. The action for the fluctuation is then

$$\begin{aligned} S = \frac{1}{2}\int dx_4 \Bigg\{ &\frac{a_0^{3/2}}{u^{14}f^3}(\partial_{x_4}\xi)^2 + \frac{a_0^{1/2}R_{\text{AdS}}^4}{u^4 f}(\partial_\mu\xi)^2 \\ &- \frac{a_0^{1/2}(u^5 + 36a_0 - 63u^{10} + 14u^{20} + 48u^{15} - 92a_0u^5 - 44a_0^{10})}{2u^{22}f^3} \Bigg\} \end{aligned} \tag{4.7}$$

and indeed this action leads to a regular e.o.m at $u(x_4 = 0) = u_0$.

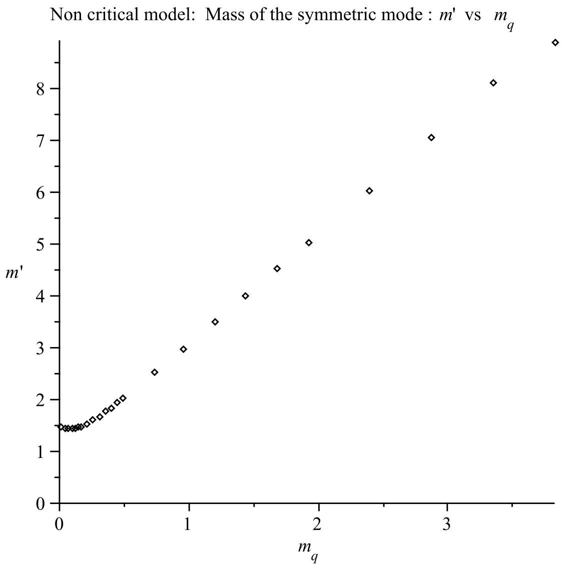
$$\begin{aligned} &\partial_x^2\xi - \left(\frac{14}{u} + \frac{15}{u^6 f}\right)u_x\partial_x\xi + \frac{u^{10}f^2R_{\text{AdS}}^4}{a_0}\eta^{\mu\nu}\partial_\mu\partial_\nu\xi \\ &+ \frac{(u^5 + 36a_0 - 63u^{10} + 14u^{20} + 48u^{15} - 92a_0u^5 - 44a_0u^{10})}{2u^8 a_0}\,\xi = 0 \end{aligned} \tag{4.8}$$

Using the shooting technique we found the eigenvalues of different modes of the fluctuation for various values of m_q^c, our finding are summarized in figures 4, 5. One can see that the masses m'_1 and m'_2 grow linearly with m_q^c as expected from (4.6). At $u_0 = u_\Lambda = 1$ we find.[9]

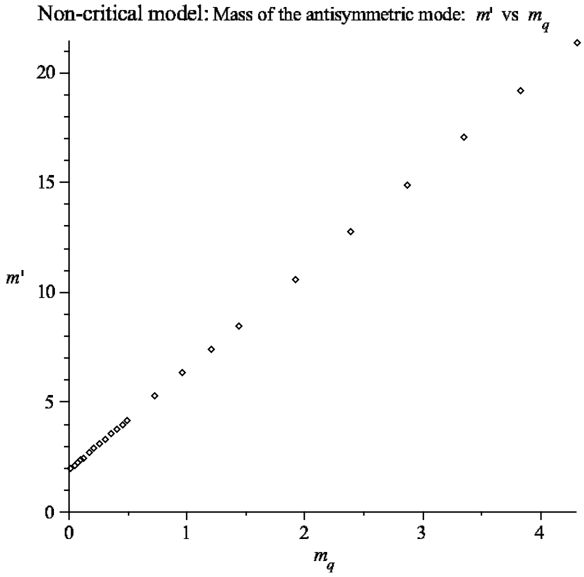
$$m_s'^2 = 1.51\,; \qquad m_{\text{as}}'^2 = 2.07. \tag{4.9}$$

which is in agreement with [5].[10] Again we also studied the dependence of the mass on the excitation number, the results are summarized in figure 6 and are:

$$m_n = 1.51 + 2.32n^{1.04} \qquad\qquad m_q^c = 0 \tag{4.10}$$



(A)



(B)

Figure 4: (A) The mass m'_1 of the lowest excited symmetric mode of the non-critical model as a function of m_q^c ($R_{\rm AdS} = u_\Lambda = 1$).

Figure 5: (B) The mass m'_2 of the lowest excited antisymmetric mode of the non-critical model as a function of m_q^c ($R_{\rm AdS} = u_\Lambda = 1$).

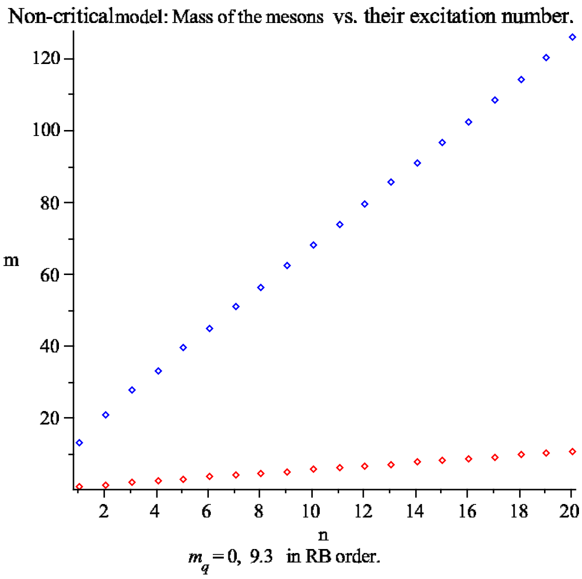
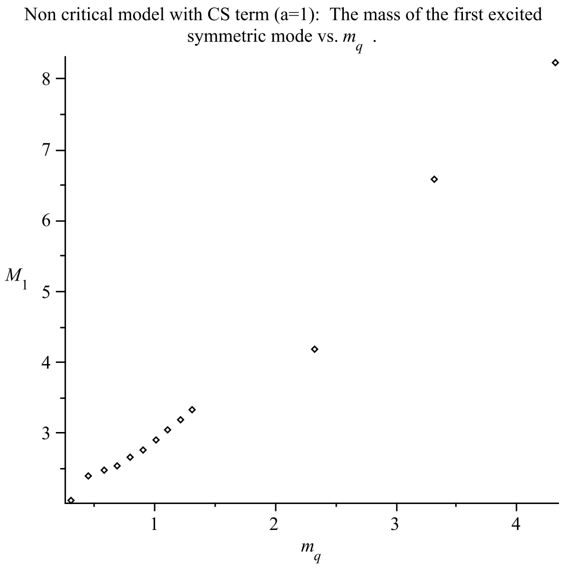


Figure 6: The tower of mesons masses m'_n in the non-critical model

$$m_n = 13.5 + 4.95 n^{1.04} \qquad\qquad m_q^c = 9.3$$

[9]Our results are for $R_{\rm AdS} = 1$.

[10]To keep contact with the results in [5] we had renormalized the masses by the factor $\frac{2}{5}$ coming from



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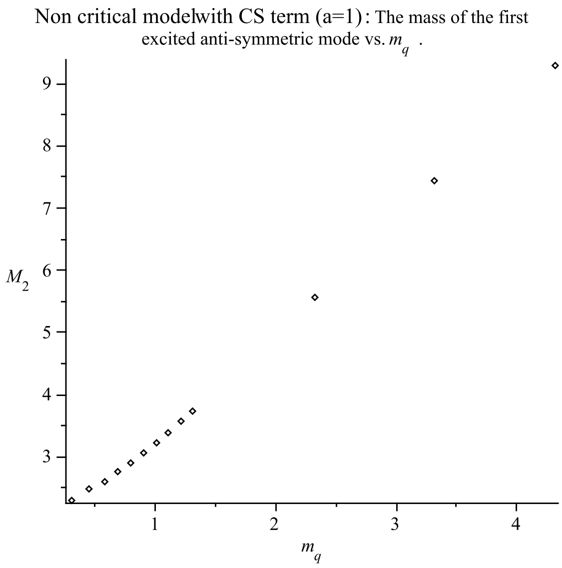


(B)

Figure 7: (A) The mass m'_1 of the lowest excited symmetric mode of the non-critical model with CS term included as a function of m_q^c .

Figure 8: (B) The mass m'_2 of the lowest excited antisymmetric mode of the non-critical model with CS term included as a function of m_q^c .

Thus we see that both in terms of the dependence on n as well as the dependence on m_q^c the scalar meson spectra admit a different behavior than that of the critical model of [1]. A similar behavior has been observed for the vector mesons in [5].

Next we consider the case where the effective action includes both the DBI and CS terms. Including now the CS term (with its full strength $\tilde{a} = 1$) the quadratic action for the fluctuation becomes

$$\begin{aligned} S = \frac{1}{2}\int dx_4 \Big\{ & \frac{B^{3/2}}{u^{14}f^3}(\partial_{x_4}\xi)^2 + \frac{B^{1/2}R_{\rm AdS}^4}{u^4 f}(\partial_\mu \xi)^2 \\ & - \frac{B^{1/2}(u^5 + 36B - 63u^{10} + 14u^{20} + 48u^{15} - 92Bu^5 - 44B^{10})}{2u^{22}f^3} - \frac{20}{\sqrt{5}}u^3\xi^2 \Big\} \end{aligned} \tag{4.11}$$

where $B = (u_0^5 f^{1/2}(u_0) - u_0^5 + u^5)^2$ and the e.o.m is then

$$\begin{aligned} & \partial_x^2\xi - \left(\frac{14}{u} + \frac{15}{u^6 f} - \frac{15u^4}{B^{1/2}}\right)u_x\partial_x\xi + \frac{u^{10}f^2R_{\rm AdS}^4}{B}\eta^{\mu\nu}\partial_\mu\partial_\nu\xi \\ & + \frac{(u^5 + 36B - 63u^{10} + 14u^{20} + 48u^{15} - 92Bu^5 - 44Bu^{10})}{2u^8 B}\,\xi + \frac{20u^3}{\sqrt{5}B^{3/2}}\xi = 0 \end{aligned} \tag{4.12}$$

With the Chern Simon taken into account the dependence of the mass squared on the excitation number is now to be read from figure 9 to be:

$$m_n = 2.07 + 5.42n^{0.75} \qquad\qquad m_q^c = 0 \tag{4.13}$$

the change of variables $u \to z$.

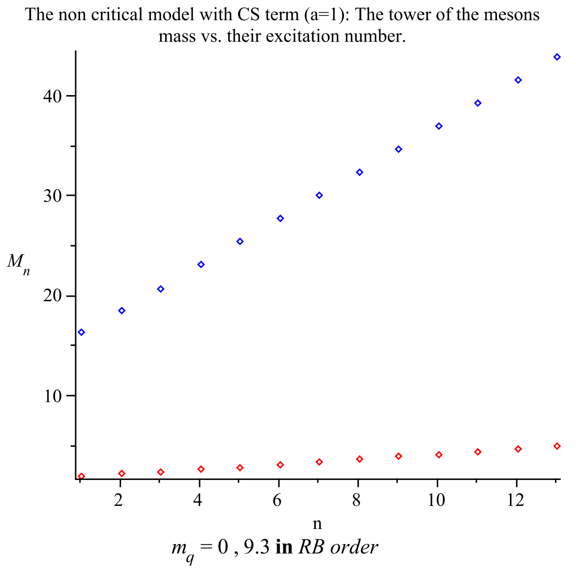


Figure 9: The tower of mesons masses m'_n in the non-critical model with CS term included

$$m_n = 16.49 + 1.01n^{0.75} \qquad\qquad m_q^c = 9.3$$

The dependence on m_q^c is described in figures 7 and 8.

5. Parity and charge conjugation

In order to compare the resulting spectra from both the critical and non-critical models, we first have to identify the "quantum numbers" of the states that correspond to the fluctuations. More explicitly we have to determine the operations in the gravity models which correspond to charge conjugation and parity transformations. In the model of [1] they were defined as follows: The *charge conjugation* operation associates with exchanging the left and right handed quarks which maps into the interchange of a $D8$ and an anti $D8$ or differently transforming $z \to -z$. *Parity* transformation in the five-dimensional space-time spanned by x_i, z where $i = 1, 2, 3$ means the following transformation $(x_i, z) \to (-x_i, -z)$.

For the generalized setup with $u_0 > u_\Lambda$ we can still define the coordinate z as follows

$$u^3 = u_0^3 + u_\Lambda z^2 \tag{5.1}$$

Note the difference with respect to (2.11) since here we take z to have dimension of length. With this definition of the z coordinate the discrete transformations of [1] remain in tact. The effective action on the probe brane has to be invariant under both parity and charge conjugation. The DBI part (2.2) is quadratic in ξ and hence cannot determine the right transformation of the fluctuation modes. The situation with the CS term is different. Recall that the CS term has the form

$$S_{\rm CS} \sim \int_{D8} F \wedge F \wedge C_5 = \int_{S^4} F \wedge F \wedge \int d^4x dz C_5 = \int_{S^4} F \wedge F \int d^4x dz \xi(x^\mu, z) \tag{5.2}$$

the last step we have used the explicit form of the C_5

$$C_5 = \xi(x^\mu, z) dx^0 \wedge \dots dx^3 \wedge dz \tag{5.3}$$

In order for this term in the action to be invariant under parity and charge conjugation it is clear that $\xi(x^\mu, z)$ has to be even under both charge conjugations and parity transformation. Now since $\xi(x,z) = \sum_n f_n(x^\mu)\xi_n(z)$ we conclude that the map between the fluctuation modes and scalar particles is the following

$$\begin{array}{lcl} \textit{symmetric } \xi_n & \rightarrow & 0^{++} \textit{ mesons} \\ \textit{antisymmetric } \xi_n & \rightarrow & 0^{--} \textit{ mesons} \end{array} \tag{5.4}$$

For the non-critical model again the DBI action does not determine the transformations of ξ under parity and charge conjugation. We have argued above based on [8] that a CS term of the form (4.1) should not be incorporated. Thus there is no way to this order to determine the transformation of ξ.

Without the constraint from the CS term we may have that ξ is even or odd under charge conjugation and parity transformations. In the latter case the assignments of (5.4) have to be reversed, namely symmetric ξ corresponds to 0^{--} and antisymmetric ξ to 0^{++}

Next we want to compare the spectra to mesons observed in nature. It is well known that scalar mesons in nature are either 0^{++} or pseudo scalars of the form 0^{-+} and there are no observed low lying mesons of the form 0^{--}. Thus there is a serious mismatch between the holographic scalar mesons extracted from models with flavor branes anti-branes of critical models and with the observed mesons in nature. We will come back to this issue in the conclusions.

6. Scalar mesons in the intermediate temperature phase

The background that corresponds to the deconfined phase, namely $T > 1/2\pi R$ is given in (1.11). As was shown in [13] this deconfined background can admit also a phase where chiral symmetry is broken, the so called "intermediate phase" We now analyze the spectrum of the scalar mesons in this phase. Since the procedure of extracting the scalar meson is identical to that of the low temperature analysis of the previous sections we present the final results for the spectra of masses. The spectra are presented in figures 10, 11 and 12. The main features that these spectra admit are the following

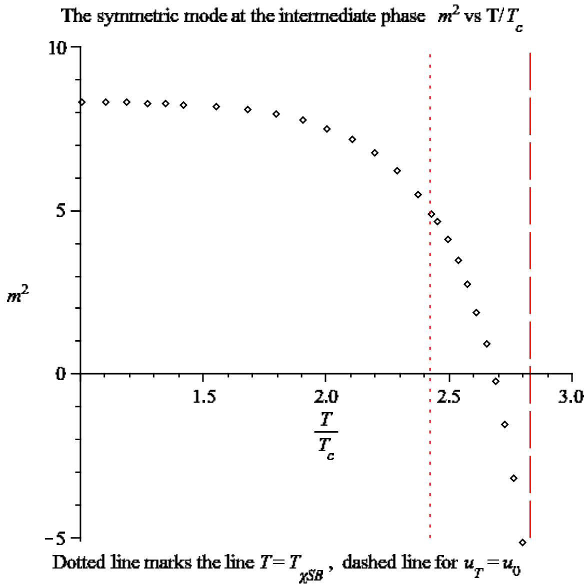
- As can be seen, at the phase transition $T = T_d$ the values of the masses are (for the values $u_T = 1, u_0 = 8$)

  $$m_s^2(T = T_d) = 8.36\,; \qquad m_{\rm as}^2(T = T_d) = 45.96 \tag{6.1}$$

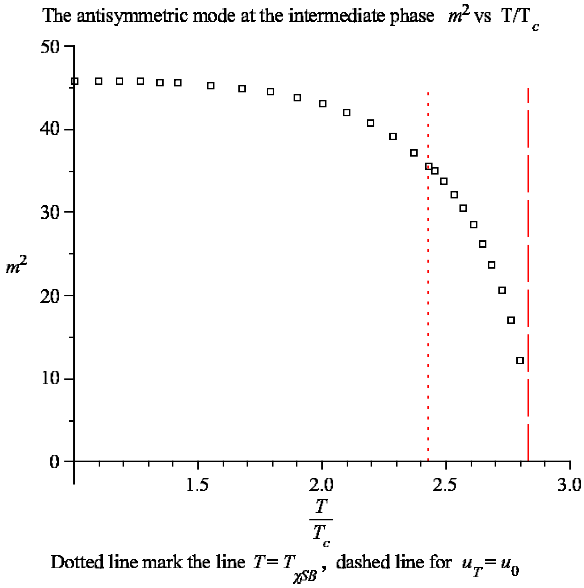
  while in the low temperature phase at the point of phase transition with $u_\Lambda \rightarrow u_T = 1$, $u_0 = 8$ the masses are

  $$m_s^2(T = T_d) = 8.40\,; \qquad m_{\rm as}^2(T = T_d) = 46.00 \tag{6.2}$$

  We see a very small jump in the masses at the transition point, the same as was seen for the vectors in [6]



(A)



(B)

Figure 10: (A) The mass squared $m_1^2(T)$ of the lowest excited symmetric mode as a function of T/T_d ($u_0 = 8, R_{D4} = 1$ and $R = 2/3$)

Figure 11: (B) The mass squared $m_2^2(T)$ of the lowest excited antisymmetric mode as a function of T/T_d ($u_0 = 8, R_{D4} = 1$ and $R = 2/3$)

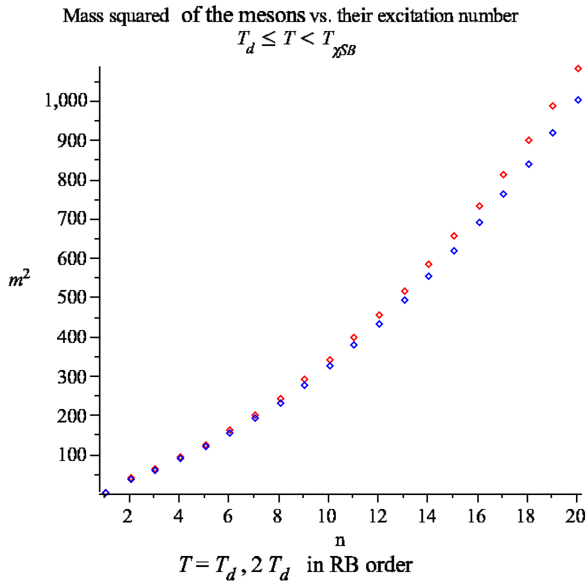


Figure 12: The tower of mesons squared mass m_n^2 in the intermediate phase

- While in the low temperature confining phase the masses of the mesons are temperature independent since the background in this phase does not depend on the temperature, the masses of the mesons do depend on the temperature in the intermediate deconfined phase. As was observed in Lattice simulations and was found also for holographic vector mesons [6], the masses decrease as a function of the temperature. The symmetric mesons decrease at the chiral symmetry phase transition temperature $T = T_{\chi SB}$ to $\sim 60\%$ percent of their values whereas the antisymmetric ones to $\sim 80\%$. This drop off is much more significant than for the vector mesons of the critical model [6].

- Note that it is only consistent to increase the temperature up to where the next phase transition occurs and chiral symmetry is restored.

  This happens at $T = T_{\chi SB}$ (for the choice $u_0 = 8$ we found that $T_{\chi SB} = 2.44T_d$), then the merged $D8$-$\bar{D}8$ breaks into a separate pair of $D8$-$\bar{D}8$. We can also see from figure 10 that if we continue to increase the temperature beyond $T_{\chi SB}$ then at some point the scalar mode becomes Tachyonic, signaling that this background is no longer stable at this temperature as indeed we know.

- Like in the low temperature we also checked the squared masses dependence on the excitation number (see figure 12). This was found to be:

$$\begin{aligned} m_n^2 &= 8.3 + 6.4n^{1.7} & T &= T_d \\ m_n^2 &= 7.6 + 6.9n^{1.65} & T &= 2T_d \end{aligned} \tag{6.3}$$

7. Conclusions

In this paper we dealt with technical problems faced in [22, 5] and succeeded to obtain the holographic mass spectra of the scalars in the low and intermediate phases of the chiral symmetry broken phase of the critical model and also of those of the non-critical. Let us summarize the results of this work and mention certain open question.

- There is a difference between the dependence of the mass of the scalar mesons on the "constituent mass parameter" m_q^c. In the ten dimensional models one finds a $m^2 \propto m_q^c$ relation (see figures 1, 2 for the first two excited modes), whereas for the non-critical model the relation is $m \propto m_q^c$ (see figures 4, 5 and 7, 8).

- Both the critical models and the non-critical one do not admit a Regge/stringy behavior of $M_n^2 \sim n$. This is not unexpected since the stringy excitations are not visible in the low energy effective field theory.

- One can compare the ratio of the low lying mesons (both vector and scalar mesons) to those observed in nature. Table 1 presents such a comparison. It is interesting to note that turning on a constituent mass m_q^c improves the ratios with respect to those for zero m_q^c.

- The holographic spectra of the critical models admit a branch of scalar mesons of type 0^{--}. These do not exist in nature. It seems to be a severe shortcoming of these holographic models. This difference cannot be attributed to the fact that we consider large N_c. It will be interesting to investigate the question of how generic

	experiment	D4-D8 at $m_q^c = 0$ / 0.38	Non-critical at $m_q^c = 0$ / 0.16
$m_{v,2}^2/m_{v,1}^2$	2.51	2.4 / -	2.8 / 2.62
$m_{v,3}^2/m_{v,1}^2$	3.56	4.3 / -	5.5 / 5.29
$m_s^2/m_{v,1}^2$	3.61	4.9 / 3.63	4.1 / 3.65
$m_{v,2}^2/m_s^2$	0.7	0.49 / 0.62	0.67 / 0.75

Table 1: A comparison with experimental data where the best fitted m_q^c is presented vs. $m_q^c = 0$ (for the critical case we have found that there is no improvement in ratios of the vectors so we left these entries empty.).

this situation is and whether one can construct a mechanism to project it out from the low lying spectra.

- The behavior of the scalar mesons at finite temperature in the intermediate phase is similar to that of the vector meson in the model of [6]. However the decrease of the mass with increasing temperature is more dramatic for the scalar mesons. It is interesting to check if a similar phenomenon occurs also in lattice simulations.

Acknowledgments

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